# Assignment 1.

This homework is due *Thursday*, September 10.

There are total 34 points in this assignment. 31 points is considered 100%. If you go over 31 points, you will get over 100% for this homework and it will count towards your course grade (but not over 115%).

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper and *credit your collaborators*. Your solutions should contain full proofs. Bare answers will not earn you much.

This assignment covers section 1.1 in Bartle–Sherbert.

(1) (Exercise 1.1.5 in textbook) Distributive laws.
  (a) [3pt] Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ by element inspection.
  (b) [2pt] Prove $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ by making a truth table.

(2) ($\sim$1.1.7) For each $n \in \mathbb{N}$, let $A_n = \{(n+1)k \mid k \in \mathbb{N}\}$. For example, $A_5 = \{6, 12, 18, \ldots\}$.
  (a) [1pt] What is $A_2 \cap A_3$?
  (b) [2pt] Determine sets $\bigcup_{n=1}^{\infty} A_n$, $\bigcap_{n=1}^{\infty} A_n$.

(3) (1.1.10) Let $f(x) = 1/x^2$, $x \neq 0$, $x \in \mathbb{R}$.
  (a) [1pt] Determine the direct image $f(E)$ where $E = \{x \in \mathbb{R} \mid 1 \leq x \leq 2\}$.
  (b) [1pt] Determine the inverse image $f^{-1}(G)$ where $G = \{x \in \mathbb{R} \mid 1 \leq x \leq 4\}$.

(4) Let $f : A \to B$ and $E, F \subseteq A$.
  (a) [3pt] (Part of 1.1.14) Show that $f(E \cup F) = f(E) \cup f(F)$.
  (b) [2pt] (Part of 1.1.14) Show that $f(E \cap F) \subseteq f(E) \cap f(F)$.
  (c) [2pt] Show that not always $f(E \cap F) = f(E) \cap f(F)$. (*Hint:* to find a counter-example, you can start by picking $E$ and $F$ that do not intersect *at all*.)
  (d) [2pt] Show that not always $f(E \setminus F) \subseteq f(E) \setminus f(F)$. (*Hint:* to find a counter-example, you can start by picking $f(E)$ and $f(F)$ that *coincide*.)

(5) (Part of 1.1.15) [3pt] Let $f : A \to B$ and $G, H \subseteq B$. Prove that $f^{-1}(G \cap H) = f^{-1}(G) \cap f^{-1}(H)$.
  Comment. Compare to 4c.

(6) (1.1.22+) Let $f : A \to B$ and $g : B \to C$.
  (a) [3pt] Show that if $g \circ f$ is injective, then $f$ is injective. Give an example that shows that $g$ need not be injective.
  (b) [3pt] Show that if $g \circ f$ is surjective, then $g$ is surjective. Give an example that shows that $f$ need not be surjective.

(7) (1.1.19)
  (a) [3pt] Show that if $f : A \to B$ is injective and $E \subseteq A$, then $f^{-1}(f(E)) = E$. Give an example that equality may fail if $f$ is not injective.
  (b) [3pt] Show that if $f : A \to B$ is surjective and $H \subseteq B$, then $f(f^{-1}(H)) = H$. Give an example to show that equality may fail if $f$ is not surjective.